

## Karl Menger and Taxicab Geometry

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In 1952 Karl Menger established a geometry exhibit at the Museum of Science and Industry of Chicago. Accompanying the exhibit was a booklet, entitled *You Will Like Geometry*, in which the term “taxicab geometry” was first used. The name has remained associated with the geometry and the geometry itself has generated interest: see the recent article (October, 1989) by Katy O. Sowell in this *MAGAZINE*. However the history seems to have fallen by the wayside along with the old geometry exhibit at the museum. The taxicab geometry is a simple way of introducing a non-Euclidean geometry to a general audience, and this indeed is done in the book *Taxicab Geometry* by Eugene F. Krause. Certainly Menger had in mind this educational value but he also saw its importance in a slightly different and more philosophical light.

Karl Menger was born in Vienna in 1902. He received his Ph.D. from the University of Vienna in 1924 under Hans Hahn. After serving as a docent at the University of Amsterdam, he returned to Vienna as a professor of geometry. At this time he joined, at Hahn’s urging, a philosophical discussion group or circle now known as the Vienna Circle. The philosophy that developed from their meetings is logical positivism, or alternatively, as many of them preferred, logical empiricism.

One of the characteristics of this group and their philosophy was an intense dislike for traditional metaphysics. It is in this context that taxicab geometry becomes important, for in this geometry the equation  $|\bar{x}| = 1$  defines a square. Thus we have a square circle which at that time was not a commonplace idea.

The metric concept was defined by Hermann Minkowski at the beginning of this century. Menger initiated the first systematic development of abstract distance geometry in 1928 with his four *Untersuchungen über allgemeine Metrik*. (See [1].) Indeed, as Menger puts it, “Square circles or round squares have haunted many diverse philosophical writers as the archetype of the impossible and the absurd; they were assigned a place near—or rather below—golden mountains, unicorns and mermaids. They have been discussed by Thomists, existentialists and linguistic philosophers as well as by Herbart, Bergson, Russell and many, many others.” [3, p. 217] Thus one of Menger’s interests in this geometry was to point out, like a true positivist, that traditional philosophy can discuss meaningless concepts at length. He concludes his article with the following: “Clearly these facts are of only limited importance for philosophers who, for their purposes, may replace the square circle by some other quasi-geometric *bête noire*. But they illustrate a new state of affairs. In the past, mathematicians have pleaded inability to associate meaning with certain metaphysical ideas. In dealing with square circles they, conversely, associate meaning with (and even give a practical interpretation to) the philosopher’s paramount example of absurdity.” [3, p. 219]

## REFERENCES

1. Leonard M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford at the Clarendon Press, 1953.
2. Eugene F. Krause, *Taxicab Geometry*, Addison Wesley, Menlo Park, NJ, 1975.
3. Karl Menger, Square Circles (The Taxicab Geometry), in *Selected Papers in Logic and Foundations, Didactics, Economics*, D. Reidel, Dordrecht: Holland/Boston:USA/London:England, 1979.
4. \_\_\_\_\_, *You Will Like Geometry Guildbook of the Illinois Institute of Technology Geometry Exhibit*, Museum of Science and Industry, Chicago, Ill, 1952.
5. Katye O. Sowell, Taxicab geometry—a new slant, *this MAGAZINE* 62 (1989), 238–248.

### On Coloring a Map

Guthrie shaded the map's every section,  
Of Albion's fair isle with affection,  
Yet his pencils grew duller,  
Drawing regions to color,  
So de Morgan he sought for direction.

A "quaternion" of color embraces,  
Maps Hamilton fiercely retraces,  
Oh conjecture so nice,  
Four colors suffice,  
Whether England or far-away places.

Heawood found the map theorem demanding,  
And the steps in a proof e'er expanding,  
With Möbius and Cayley,  
He considered it daily,  
Still the problem defied understanding.

Said Kempe, "my color connection,  
Is a 'proof' which demands some perfection."  
Greater fame it thus gained,  
As the problem remained,  
Till Appel and Haken's correction.

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*Note:* The origin of the four-color map problem poses an unanswered question. Some attribute the remarkable conjecture to Francis Guthrie of the nineteenth century. His discovery stemmed from coloring a map of England. A succession of great mathematicians holds a prominent place in the continuing story.